CALCULATING ORBIT ERRORS IN MAIN INJECTOR USING TWO OR ONE BPM/CELL CORRECTION SCHEME

Fady A. Harfoush

December 3, 1992

1 Introduction

This report summarizes calculations done about two years ago to look at two different orbit correction schemes for the Main Injector. The first correction scheme considers information from the two beam position monitors (bpm) placed at focussing and defocussing quadrupoles while the second scheme considers information from the bpm at only defocussing quadrupoles. The obvious advantages of the one bpm per cell compared to the two bpms per cell are less cost, and less noise introduced in the system from the electronics. However the one bpm per cell scheme provides less information about the beam and can be a problem in detecting any instabilities in the beam. This calculation follows Chao's and Peggs paper [1] and is based on a simple FODO cell with one corrector per plane and two bpms per plane. In each case the rms correctors strength and rms orbit errors are calculated using the method of least square minimization. To better qualify the correction scheme orbit errors at focussing and defocussing quadrupoles are calculated separately. Different types of orbit error sources are considered. These errors can be due to quadrupoles and monitors misalignements, dipole roll error in the vertical plane, and dipole field error in the horizontal plane. Results obtained for the correctors strength are in agreement with Rod Gerig's results [2].

2 Theory

Calculations in this paper are based on the theory of orbit correction using localized orbit bumps [3]. Consider a simple lattice made of FODO (Focussing Quad-Drift-Defocussing Quad-Drift) cells. Let the vector $\vec{x}_d(n)$ describes the initial distorted orbit where n represents the number of monitors. An orbit correction vector $\vec{x}_c(n)$ is added to the vector $\vec{x}_d(n)$ such that the norm squared of the vector $\| \vec{x}_d + \vec{x}_c \|^2$ is minimized. This method is known as the method of least squares. The orbit correction vector $\vec{x}_c$ is related to the corrector strength vector by:

$$\vec{x}_c = T\vec{\theta} \tag{1}$$

where T is an $n \times m$ matrix and m is equal to the number of correctors. In general the elements of the matrix T are given by:

$$T_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi\nu} \cos \nu(\pm\pi + \phi_i - \phi_j) \tag{2}$$

where ν is the betatron wave number, β is the betratron frequency and $\nu\phi$ is the betratron phase. The plus sign in front of π is used for the case where $\phi_i < \phi_j$ and the minus sign for the case $\phi_i > \phi_j$. With localized orbit bumps the elements of matrix T take the simpler form of:

$$T_{i,i-1} = \frac{1}{\sqrt{1 + d_{i,i-1}^2 + d_{i,i+1}^2}} d_{i,i-1} \tag{3}$$

$$T_{i,i} = \frac{1}{\sqrt{1 + d_{i,i-1}^2 + d_{i,i+1}^2}} \tag{4}$$

$$T_{i,i+1} = \frac{1}{\sqrt{1 + d_{i,i-1}^2 + d_{i,i+1}^2}} d_{i,i+1} \tag{5}$$

$$\tag{6}$$

where:

$$d_{i,j} = \sqrt{\frac{\beta_j}{\beta_i}} \frac{\sin(\phi_{i-1} - \phi_{j-1})}{\sin(\phi_i - \phi_{j-1})} \tag{7}$$

The closed bump is located at the $i-th$ monitor. The least square minimization condition leads to a corrector stength vector given by:

$$\vec{\theta} = -(\tilde{T}T)^{-1}\tilde{T}\vec{x}_d \tag{8}$$

2.1 Two BPMs versus One BPM Correction Scheme

With the two bpms per cell correction scheme bpm at each focussing and defocussing quadrupole provides readings in both planes horizontal and vertical. For each plane, therefore, there are one corrector and two bpms per cell. The one bpm per cell correction scheme has for each plane one corrector and one bpm per cell. A descriptive representation of each case is given in the Fig 1.

The matrix T described in the previous section was derived using information from the two bpms per cell per plane. With the one bpm per plane the information from the bpms at defocussing quadrupoles is not available for the

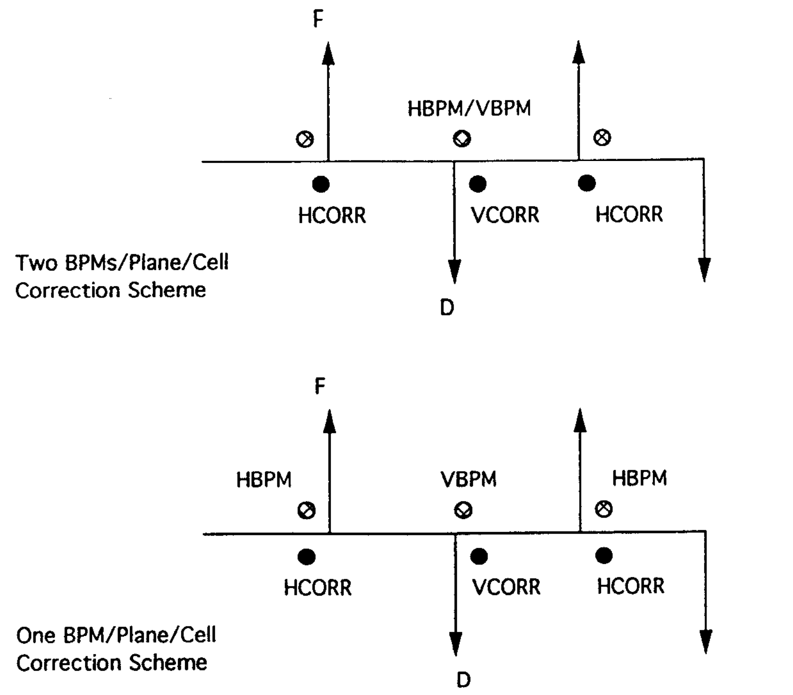

Figure 1: Illustrating two bpms and one bpm correction schemes

calculation of the beam bump corrector strength in the horizontal plane. Instead of matrix T the strength of the corrector vector is now calculated using a new matrix T' that has the form:

$$T'_{i,i} = \frac{1}{\sqrt{1 + d^2_{i,i-1} + d^2_{i,i+1}}} \tag{9}$$

$$T'_{i,j} = 0.\text{eslsewhwere} \tag{10}$$

where $d_{i,j}$ is as described earlier. The information from the bpm at defocussing quadrupoles has been omitted by assigning zero to that element. The case when information from focussing quadrupoles, where correctors are located, is omitted does not make sense and will lead to a singularity problem. The orbit correction vector $\vec{x}_c$ is now calculated using the new matrix T'.

3 Results

Different types of orbit error sources, as mentionned in the introduction, are considered. The following typical rms values for the Main Injector are selected:

- Quadrupole displacements (both focussing and defocussing) of 0.250 mm (rms).
- Beam monitor displacement of 0.250 mm (rms).
- Dipole field error of 5×10^{-4}.
- Dipole roll error of 0.250 mrad (rms).

The cell length is taken as 34.3 m, the phase advance per cell as 90^o, the number of dipoles per half cell as 2, and the total number of dipoles as 300 resulting in a bending angle $\theta_b = 41.8\ mrad$ per half cell. A program called ORBIT-2BPM/1BPM was developed to compute the rms corrector strength for each unit error source.

3.1 The Two BPMS Case

Each source of error will contribute to the total rms corrector strength and the total rms corrected orbit according to the following equations:

$$< \theta^2 >^{1/2} = \{(1.00)^2 < (\delta x_{df}/f)^2 > +(0.389)^2 < (\delta x_{qd}/f)^2 > \tag{11}$$
$$+2(0.742)^2 < \delta_\theta^2 > +(0.02112)^2 < \delta x_{bpm}^2 > +(0.00775)^2 < \delta x_{bpm}^2 >\}^{1/2}$$

$$< x^2 >^{1/2} = \frac{1}{\sqrt{2}} \{(0.00)^2 < (\delta x_{df}/f)^2 > +(4.66)^2 < (\delta x_{qd}/f)^2 > \tag{12}$$
$$+2(4.66)^2 < \delta_\theta^2 > +(0.370)^2 < \delta x_{bpm}^2 > +(0.929)^2 < \delta x_{bpm}^2 >\}^{1/2}$$

The factor $\frac{1}{\sqrt{2}}$ appearing in (12) is because the total number of bpms is twice the total number of correctors and the total number of focussing or defocussing quadrupoles [1]. The constant coefficients in the above equations represent the squared sum of all the corrector strength for each unit of error and the squared sum of the corrected orbit for each unit of error type considered. To better evaluate the correction scheme the rms error orbit and corrected orbit are separated in two parts: a summation over bpms at focussing quads only and a summation over bpms at defocussing quads only. Results are shown in Tables 1 and 2 [2].

3.2 The One BPM Case

Here we consider information from only one bpm per plane. The coefficients of the equations describing the total rms corrector strength and the total rms corrected orbit are now given by:

$$< \theta^2 >^{1/2} = \{(1.00)^2 < (\delta x_{df}/f)^2 > +(0.4142)^2 < (\delta x_{qd}/f)^2 > \tag{13}$$
$$+2(0.7654)^2 < \delta_\theta^2 > +(0.02415)^2 < \delta x_{bpm}^2 > +(0.0000)^2 < \delta x_{bpm}^2 >\}^{1/2}$$

$$< x^2 >^{1/2} = \frac{1}{\sqrt{2}} \{(0.00)^2 < (\delta x_{df}/f)^2 > +(5.0231)^2 < (\delta x_{qd}/f)^2 > \tag{14}$$
$$+2(5.0231)^2 < \delta_\theta^2 > +(0.4142)^2 < \delta x_{bpm}^2 > +(1.0000)^2 < \delta x_{bpm}^2 >\}^{1/2}$$

Results are given Tables 3 and 4

[1] In [1] this factor was omitted

[2] rms in tables stands for $\sqrt{\sum_1^N x_i^2}$

4 RMS Correctors Strength and Orbit Distortion

The rms corrector strength and orbit distortion for each correction scheme are calculated using equations (13) and (14). The rms corrector strength for each type of error source is calculated separately and the total result given. This allow us to determine the dominant source of error in calculating the corrector strength. Results similar to the previous case are shown in Tables 5 and 6.

5 Conclusions

The following observations can be finally made.

- The two BPMs/cell correction scheme gives a better orbit correction at focussing quadrupoles (where correctors are located) than at defocussing quadrupoles
- The one BPM/cell gives a perfect orbit correction at focussing quadrupoles
- At defocussing quadrupoles the one BPM/cell correction scheme is worse by about 20% than the two BPMs/cell scheme
- Overall the rms closed orbit distortion with one BPM/cell correction scheme is worse by about 9.5% than the two BPMs/cell scheme.

References

[1] Alexander W. Chao and Steve Peggs, *SSC-48*, October 1985.

[2] Rod Gerig, *MI-0028*, August 1990.

[3] Steve Peggs, *Ph. D. Thesis*, Cornell University, 1981.

Source of Error	$\theta_{rms}(mrad)$	$X_{rms}^{error}(mm)$	$X_{rms}^{corrected}(mm)$
1 mrad angular kick @ FQ	1.000	208.570	0.000
1 mrad angular kick @ DQ	0.389	86.287	1.667
1 mrad kick error @ FQ-DQ	0.742	159.79	1.667
1 mrad kick error @ DQ-FQ	0.742	159.79	1.667
1 mm BPM displacement @ FQ	0.0211	1.00	0.164
1 mm BPM displacement @ DQ	0.00775	0.00	0.332

Table 1: Two BPMs case: Summing over BPMs @ focussing quads

Source of Error	$\theta_{rms}(mrad)$	$X_{rms}^{error}(mm)$	$X_{rms}^{corrected}(mm)$
1 mrad angular kick @ FQ	0.0	86.093	0.000
1 mrad angular kick @ DQ	0.0	35.340	4.358
1 mrad kick error @ FQ-DQ	0.0	65.768	4.358
1 mrad kick error @ DQ-FQ	0.0	65.768	4.358
1 mm BPM displacement @ FQ	0.0	0.00	0.332
1 mm BPM displacement @ DQ	0.0	1.00	0.867

Table 2: Two BPMs case: Summing over BPMs @ defocussing quads

Source of Error	$\theta_{rms}(mrad)$	$X_{rms}^{error}(mm)$	$X_{rms}^{corrected}(mm)$
1 mrad angular kick @ FQ	1.000	208.572	0.000
1 mrad angular kick @ DQ	0.414	86.287	0.000
1 mrad kick error @ FQ-DQ	0.765	159.791	0.000
1 mrad kick error @ DQ-FQ	0.765	159.791	0.000
1 mm BPM displacement @ FQ	0.0241	1.000	0.000
1 mm BPM displacement @ DQ	0.000	0.000	0.000

Table 3: One BPM case: Summing over BPMs @ focussing quads

Source of Error	$\theta_{rms}(mrad)$	$X_{rms}^{error}(mm)$	$X_{rms}^{corrected}(mm)$
1 mrad angular kick @ FQ	0.0	86.093	0.000
1 mrad angular kick @ DQ	0.0	35.43	5.023
1 mrad kick error @ FQ-DQ	0.0	65.768	5.023
1 mrad kick error @ DQ-FQ	0.0	65.768	5.023
1 mm BPM displacement @ FQ	0.0	0.00	0.414
1 mm BPM displacement @ DQ	0.0	1.00	1.000

Table 4: One BPM case: Summing over BPMs @ defocussing quads

Source of Error	Horizontal Corrector (μrad)	Vertical Corrector
Misalignment of FQ	20.6154	20.6154
Misalignment of DQ	8.0399	8.0399
Dipole roll	0.0000	7.78525
Dipole field error	15.5705	0.0000
Misalignment of BPMS	5.6234	5.6234
Total strength	27.6351	24.1219

Table 5: Two BPMs case: Correctors strength

Source of Error	Horizontal Distortion (mm)	Vertical Distortion
Misalignment of FQ	0.0000	0.0000
Misalignment of DQ	0.09607	0.09607
Dipole roll	0.0000	0.04869
Dipole field error	0.09739	0.0000
Misalignment of BPMS	0.2499	0.2499
Total distortion	0.284975	0.272207

Table 6: Two BPMs case: Orbit distortion

Source of Error	Horizontal Corrector (μrad)	Vertical Corrector
Misalignment of FQ	20.6154	20.6154
Misalignment of DQ	8.5388	8.5388
Dipole roll	0.0000	7.9984
Dipole field error	15.9969	0.0000
Misalignment of BPMS	6.0375	6.0375
Total strength	28.1115	24.4608

Table 7: One BPM case: Correctors strength

Source of Error	Horizontal Distortion (mm)	Vertical Distortion
Misalignment of FQ	0.0000	0.0000
Misalignment of DQ	0.1035	0.1035
Dipole roll	0.0000	0.05249
Dipole field error	0.1049	0.0000
Misalignment of BPMS	0.2706	0.2706
Total distortion	0.308168	0.294451

Table 8: One BPM case: Orbit distortion